ID-Based Encryption for Complex Hierarchies with Applications to Forward Security and Broadcast Encryption*

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ABSTRACT

A forward-secure encryption scheme protects secret keys from exposure by evolving the keys with time. Forward security has several unique requirements in hierarchical identity-based encryption (HIBE) scheme: (1) users join dynamically; (2) encryption is joining-time-oblivious; (3) users evolve secret keys autonomously.

We present a scalable forward-secure HIBE (fs-HIBE) scheme satisfying the above properties. We also show how our fs-HIBE scheme can be used to construct a forward-secure public-key broadcast encryption scheme, which protects the secrecy of prior transmissions in the broadcast encryption setting. We further generalize fs-HIBE into a collusion-resistant multiple hierarchical ID-based encryption scheme, which can be used for secure communications with entities having multiple roles in role-based access control. The security of our schemes is based on the bilinear Diffie-Hellman assumption in the random oracle model.

Categories and Subject Descriptors

E.3 [**Data Encryption**]: Public-Key Cryptosystems

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General Terms

Algorithns, Security, Theory

Keywords

Forward security, ID-Based Encryption, Broadcast Encryption

1. INTRODUCTION

The idea of an identity-based encryption (IBE) scheme is that an arbitrary string can serve as a public key. The main advantage of this approach is to largely reduce the need for public key certificates and certificate authorities, because a public key is associated with identity information such as a user's email address. A first scheme for identity-based encryption (BF-IBE) was based on the bilinear Diffie-Hellman assumption in the random oracle model by Boneh and Franklin [8]. In IBE schemes private key generator (PKG) is responsible for generating private keys for all users, and therefore is a performance bottleneck for organizations with large number of users. Hierarchical identity-based encryption (HIBE) schemes [6, 18, 22] were proposed to alleviate the workload of a root PKG by delegating private key generation and identity authentication to lower-level PKGs. The organization of PKGs and users forms a hierarchy that is rooted by the root PKG. Gentry and Silverberg [18] extended BF-IBE scheme and presented a fully scalable hierarchical identity-based encryption (GS-HIBE) scheme. Later, a HIBE construction with a weaker notion of security was given by Boneh and Boyen [6]. Most recently, new IBE and HIBE constructions that can be proved to have the full security without the random oracle model [7, 30] were given.

Due to the inherent key-escrow property[1], the standard notion of HIBE security crucially depends on secret keys remaining secret. Key exposure is a realistic threat over the

[1]A PKG knows the private keys of its child nodes.

lifetime of such a scheme. To mitigate the damage caused by the exposure of secret key information in HIBE, one way is to construct a forward-secure hierarchical identity-based encryption (fs-HIBE) scheme that allows each user in the hierarchy to refresh his or her private keys periodically while keeping the public key the same. A forward-secure public-key encryption scheme has recently been presented by Canetti, Halevi and Katz [10]. But surprisingly, a practical fs-HIBE scheme has several unique requirements that cannot be achieved by trivial combinations of the existing fs-PKE schemes [10, 23] and HIBE scheme [6, 18].

Apart from being interesting on its own, fs-HIBE is a useful tool that lends itself to several applications. One such application is the implementation of forward secrecy for public-key broadcast encryption. While forward secrecy is an important requirement in any context, it is especially needed for broadcast encryption [14, 16, 25, 31]. This is because by design an adversary can freely listen to any broadcast and store it. Then, should the adversary ever succeed in recovering *any* user's secret key, she will manage to decrypt all past broadcasts that such user was authorized to receive *unless* we have forward secrecy.

Below, we discuss the notion of forward security for HIBE in more detail, and then explain why it cannot be trivially achieved by existing techniques such as a combination of fs-PKE [10] and HIBE [6, 18] schemes.

1.1 Forward Security

The central idea of forward secrecy is that the compromise of long-term keys does not compromise past session keys and therefore past communications. This notion was first proposed by Günther [17] and later by Diffie *et al.* [11] in key exchange protocols. The notion of non-interactive forward security was proposed by Anderson [2] in 1997 and later formalized by Bellare and Miner [3], who also gave a forward-secure signature scheme followed by a line of improvement [1, 26]. In this model, secret keys are updated at regular intervals throughout the lifetime of the system; furthermore, exposure of a secret key corresponding to a given interval does not enable an adversary to break the system (in the appropriate sense) for any prior time period. The model inherently cannot prevent the adversary from breaking the security of the system for any subsequent time period. Bellare and Yee [5] provided a comprehensive treatment of forward security in the context of private key based cryptographic primitives.

The first forward-secure public-key encryption (fs-PKE) scheme was given by Canetti, Halevi, and Katz [10] based on the Gentry-Silverberg HIBE [18] scheme. The fs-PKE scheme constructs a binary tree, in which a tree node corresponds to a time period and has a secret key. Children of a node w are labeled $w0$ and $w1$, respectively. Given the secrets corresponding to a prefix of a node representing time t, one can compute the secrets of time t. In order to make future keys computable from the current key, the secrets associated with a prefix of a future time are stored in the current key. After the key for the next time period is generated, the current decryption key is erased. The state-of-the-art fs-PKE scheme [10] is based on the decisional Bilinear Diffie-Hellman assumption [8] in the standard model. Canetti, Halevi and Katz also gave a more efficient scheme in the random oracle model [10].

1.2 Requirements of an fs-HIBE Scheme

Intuitively, forward security in a HIBE scheme implies that compromise of the current secret key of a user only leads to the compromise of the user and his descendants' subsequent communications. We will give a formal definition of security in Section 2.3. Our design of a forward-secure HIBE scheme also takes system properties such as scalability and efficiency into consideration. This is essential in the management of large scale distributed systems. Below, we define the requirements for a scalable forward-secure HIBE scheme.

- New users should be able to join the hierarchy and receive secret keys from their parent nodes *at any time*.
- Encryption is *joining-time-oblivious*, which means that the encryption does not require knowledge of when a user or any of his ancestors joined the hierarchy. The sender can encrypt the message as long as he knows the current time and the ID-tuple of the receiver, along with the public parameters of the system.
- The scheme should be forward-secure.
- Refreshing secret keys can be carried out *autonomously*, that is, users can refresh their secret keys on their own to avoid any communication overhead with any PKG.

Surprisingly, the design of an fs-HIBE scheme that fulfils the above system requirements turns out to be non-trivial, despite the fact that both HIBE [18] scheme and fs-PKE [10] scheme are known. Intuitive combinations of the two schemes fail to achieve all the desired system features. Next, we explain why this is the case.

1.3 Some Forward-Secure HIBE Attempts

In this section, we make three simple forward-secure HIBE constructions based on HIBE scheme [18] and fs-PKE scheme [10], and explain why these naive schemes do not satisfy the requirements of a practical fs-HIBE scheme.

Scheme I Consider a scheme based on the HIBE [18] scheme. The user with a given ID tuple $(ID_1, \ldots, ID_h)$ maintains two sub-hierarchies (subtrees): the time subtree that evolves over time for forward security (as in fs-PKE [10]), and the ID subtree to which other nodes are added as children join the hierarchy. To encrypt a message for this user at time t, use the HIBE with identity $(ID_1, \ldots, ID_h, t)$. The user can decrypt this message using HIBE decryption, using the fact that he knows the key from the time subtree. The user's children are added to the hierarchy into the ID subtree.

The problem with this scheme is combining dynamic joins with forward security. Suppose a user never erases the secret key corresponding to the root of his ID subtree. Then should this key ever be exposed, the forward secrecy of his children is compromised. On the other hand, if this secret key is ever erased, then no nodes can be added as children of $(ID_1, \ldots, ID_h)$ in the hierarchy, and so this scheme will not support dynamic joins.

The lesson we learn from this failed scheme is that all keys must be evolved together.

Scheme II Let us try to repair Scheme I by making sure that the key from which children's keys are derived is also evolving over time. In Scheme II, the public key of a user consists of alternating ID-tuples and time strings, which is

referred to as an *ID-time-tuple*. The private key of a user serves three purposes: decryption, generating private keys for new children, and deriving future private keys of the user. The public key of a newly joined child is the parent's ID-time-tuple appended with the child's ID. That key is in turn used for generating keys for lower-level nodes further down the hierarchy. For example, if Alice joins Bob, the root, at time (*January, Week 1*) and Eve joins Alice at time (*January, Week 2*), Eve's public key is (Bob, *January, Week 1*, Alice, *January, Week 2*, Eve). Encrypting a message to Eve requires the sender to know when Eve and all her ancestors joined the system. Therefore Scheme II is not joining-time-oblivious.

The lesson we learn from the failed Scheme II is that the keys must evolve in a way that is transparent to the encryption algorithm.

Scheme III In our final unsuccessful attempt, Scheme III, a user adds a child to the hierarchy by giving him or her secret keys that depend both on the current time and on the child's position in the hierarchy. This is achieved by requiring that messages may only be decrypted by those who know two keys: one corresponding to the current time and the other corresponding to their positions in the hierarchy. Each user autonomously evolves his time key, and gives his newly joined children his time key in addition to their ID keys.

It is easy to see that this scheme is *n*ot forward-secure. An adversary who joins the hierarchy at the beginning of time can corrupt a user at any future time and obtain his or her ID key. Moreover, this adversary can derive any past time key (because he joined at the beginning of time). Thus, this adversary may decrypt any past message addressed to the exposed user.

Comparisons All the above trivial approaches fail. Therefore, constructing a forward-secure hierarchical ID-based encryption scheme that is both secure and scalable is not so straightforward. Our implementation overcomes the problems existing in naive combinations of the two schemes.

1.4 Our Contributions

We make several contributions in this paper. First, we present a scalable and joining-time-oblivious forward-secure hierarchical identity-based encryption scheme that allows keys to be updated autonomously. Second, we show how our fs-HIBE scheme can be used to obtain a forward-secure public-key broadcast encryption (fs-BE) scheme. Third, we generalize our fs-HIBE scheme and discuss its application in secure communications with entities having multiple roles in role-based access control (RBAC) [28].

1.4.1 Forward-Secure HIBE Scheme

Our fs-HIBE protocol is based on the HIBE scheme by Gentry and Silverberg [18] and forward-secure public-key encryption (fs-PKE) [10] scheme due to Canetti, Halevi and Katz. It satisfies the requirements of dynamic joins, joining-time-obliviousness, forward security, and autonomous key updates.

A HIBE scheme involves only one hierarchy, whereas an fs-HIBE scheme has two hierarchies: ID and time. Each (ID-tuple, time) pair can be thought of as a point on the two-dimensional grid as follows. On the x-axis, we start with the identity of the root Public Key Generator in the ID hierarchy (e.g. Hospital), then in position (1,0) we have the identity of the first-level PKG (e.g. ER). In position (2,0) there is the identity of the second level PKG (e.g. Doctor), and in position (3,0) there may be another PKG or an individual user (e.g. Bob). Thus the x-axis represents an ID-tuple, for example (Hospital, ER, Doctor, Bob). Similarly, the y-axis represents the time. Divide a duration of time into multiple time periods and arrange them as leaf nodes of a tree. Internal nodes of the tree represent the time spans associated with their child nodes. Then, the origin of the grid corresponds to the root of the time hierarchy (e.g. 2004). In position (0, 1) we have the first level of the time hierarchy (e.g. January), and in position (0, 2) there is the next level of time hierarchy (e.g. Week 1). Thus a time period can be expressed as a tuple on the y-axis, for example (2004, January, Week 1).

In an fs-HIBE scheme, the secret key of an (ID-tuple, time) pair is associated with some path on the grid. For each grid point on that path, there is a corresponding element in this secret key. Such a path (secret key) is *not* joining-time-oblivious: it depends on when the user, as well as the nodes higher up, join the system. However, when encrypting, the sender does not have to know the path. What is non-trivial here is that, the path (secret key) and ciphertext of our fs-HIBE scheme are designed in such a way that we do not need to come up with a separate ciphertext for each possible path in order to achieve joining-time-obliviousness.

Our fs-HIBE scheme has collusion resistance and chosen ciphertext security in the random oracle model [4] assuming the difficulty of the Bilinear Diffie-Hellman problem [8, 10, 18], provided that the depths of the ID hierarchy and time hierarchy are bounded by constants. The formal definitions of the scheme are given in the paper and the proofs are in the full paper [32]. The complexities of various parameters in our fs-HIBE scheme are summarized in Table 1 and are discussed in Section 6.

1.4.2 Forward-Secure Broadcast Encryption Scheme

We show how our fs-HIBE scheme can be used to construct a scalable forward-secure public-key broadcast encryption (fs-BE) scheme, which protects the secrecy of prior transmissions. A broadcast encryption (BE) [12, 13, 19, 20, 24, 27] scheme allows content providers to securely distribute digital contents to a dynamically changing user population. Each active user is issued a distinct secret key when he joins the system, by a trusted center. In comparison with the symmetric-key setting, a public-key BE scheme of [12] has a single public key associated with the system, which allows the distribution of the broadcast workload to untrusted third parties.

In a scalable forward-secure public-key broadcast encryption (fs-BE) scheme, users should be able to update their secret keys autonomously, and the trusted center should allow users to dynamically join the broadcast system at any time while achieving forward security. In addition, each content provider does not need to know when each user joins the system in order to broadcast the encrypted contents. The encryption algorithm of an fs-BE scheme should only depend on the current time and the set of authorized users, and thus be joining-time-oblivious. Applying our fs-HIBE to the public-key BE scheme [12] yields such an fs-BE scheme.

1.4.3 Multiple Hierarchical ID-Based Encryption

We further generalize our forward-secure hierarchical ID-based encryption scheme into a collusion-resistant multiple hierarchical identity-based encryption (MHIBE) scheme, and describe its application in secure communications with individuals who have multiple roles in role-based access control (RBAC) [28]. In large-scale organizations, a user may own multiple identities, each of which is represented by an ID-tuple. In MHIBE, a message can be encrypted under multiple ID-tuples (identities) and can be decrypted only by those who have *all* the required identities. The collusion-resistant property cannot be achieved using separate HIBE schemes.

1.5 Outline of the Paper

The rest of the paper is organized as follows. In Section 2 we give definitions for fs-HIBE scheme and its security. In Section 3, we first recall the bilinear Diffie-Hellman assumption [8], and then give the construction of an fs-HIBE scheme and analyze the security. In Section 4, we show how an fs-HIBE scheme can be used to add forward secrecy to a public-key broadcast encryption scheme. In Section 5, we describe the multiple hierarchical ID-based encryption scheme. The complexity analysis is given in Section 6.

2. FORWARD-SECURE HIBE (FS-HIBE)

This section defines the notion of forward secrecy for HIBE scheme and the related security.

In an fs-HIBE scheme, secret keys associated with an ID-tuple are evolved with time. At any time period i an entity joins the system (hierarchy), its parent node computes its decryption key corresponding to time period i and other values necessary for the entity to compute its own future secret keys. Once the newly joined entity receives this secret information, at the end of each period it updates its secret key and erases the old key. During time period i, a message is encrypted under an ID-tuple and the time i. Decryption requires the secret key of the ID-tuple at time i.

2.1 Notations

Time Period: As usual in forward-secure public-key encryption [10] scheme, we assume for simplicity that the total number of time periods N is a power of 2; that is $N = 2^l$.
ID-tuple: An entity has a position in the hierarchy, defined by its tuple of IDs: $(\mathrm{ID}_1, \ldots, \mathrm{ID}_h)$. The entity's ancestors in the hierarchy are the users / PKGs whose ID-tuples are $\{(\mathrm{ID}_1, \ldots, \mathrm{ID}_i) : 1 \leq i < h\}$. ID_1 is the ID for the root PKG.

Keys: There are two types of keys: $sk_{w,(\mathrm{ID}_1,\ldots,\mathrm{ID}_h)}$ and $SK_{i,(\mathrm{ID}_1,\ldots,\mathrm{ID}_h)}$. The node key $sk_{w,(\mathrm{ID}_1,\ldots,\mathrm{ID}_h)}$ is the key associated with some prefix w of the bit representation of a time period i and a tuple $(\mathrm{ID}_1, \ldots, \mathrm{ID}_h)$. $SK_{i,(\mathrm{ID}_1,\ldots,\mathrm{ID}_h)}$ denotes the key associated with time i and an ID-tuple $(\mathrm{ID}_1, \ldots, \mathrm{ID}_h)$. It consists of the following sk keys $(sk_{i,(\mathrm{ID}_1,\ldots,\mathrm{ID}_h)}, \{sk_{w1,(\mathrm{ID}_1,\ldots,\mathrm{ID}_h)} : w0 \text{ is a prefix of } i\})$. When this causes no confusion, we denote the keys as $sk_{w,h}$ and $SK_{i,h}$, respectively.

2.2 fs-HIBE: Syntax

Forward-secure Hierarchical ID-Based Encryption (fs-HIBE) scheme: an fs-HIBE scheme is specified by five algorithms: Root Setup, Lower-level Setup, Update, Encrypt, and Decrypt:

Root Setup: The root PKG takes a security parameter k and the total number of time periods N, and returns $params$ (system parameters) and the initial root key $SK_{0,1}$. The system parameters include a description of the message space $\mathcal{M}$ and the ciphertext space $\mathcal{C}$. The system parameters will be publicly available, while only the root PKG knows the initial root key.
Lower-level Setup: This algorithm is run by the parent of a newly joined child at time i to compute the child's private key. During a time period i, a lower-level entity (user or lower-level PKG) joins in the system at level h. Its parent at level $h-1$ computes the entity's key $SK_{i,h}$ associated with time period i. The inputs are the parent's private key $SK_{i,h-1}$, time i, and the ID-tuple of the child. (Note that the functionality of our Lower-level Setup includes the functionality of both the Lower-level Setup and the Extraction algorithm in the HIBE [18] scheme. This simplifies the protocol without any loss of generality.)
Update: During the time period i, an entity (PKG or individual) with ID-tuple $(\mathrm{ID}_1, \ldots, \mathrm{ID}_h)$ uses $SK_{i,h}$ to compute his key $SK_{(i+1),h}$ for the next time period $i+1$, and erases $SK_{i,h}$.
Encrypt: A sender inputs $params$, the index i of the current time period, $M \in \mathcal{M}$ and the ID-tuple of the intended message recipient, and computes a ciphertext $C \in \mathcal{C}$.
Decrypt: During the time period i, a user with the ID-tuple $(\mathrm{ID}_1, \ldots, \mathrm{ID}_h)$ inputs $params$, $C \in \mathcal{C}$, and its secret key $SK_{i,h}$ associated with time period i and the ID-tuple, and returns the message $M \in \mathcal{M}$.
Encryption and decryption must satisfy the standard consistency constraint, namely when $SK_{i,h}$ is the secret key generated by algorithm Lower-level Setup for ID-tuple $(\mathrm{ID}_1, \ldots, \mathrm{ID}_h)$ and time period i, then: $\forall M \in \mathcal{M}$, decryption of the ciphertext C with $params$ and the key $SK_{i,h}$ yields the message M, where C is the result of the encryption of M under time i and $(\mathrm{ID}_1, \ldots, \mathrm{ID}_h)$.

2.3 fs-HIBE: Security

We allow an attacker to make *lower-level setup queries*. Also, we allow the adversary to choose the time period and the identity on which it wishes to be challenged. Notice that an adversary may *choose* the time period and the identity of its targets adaptively or nonadaptively. An adversary that chooses its targets adaptively first makes lower-level setup queries and decryption queries, and then chooses its targets based on the results of these queries. A nonadaptive adversary, on the other hand, chooses its targets independently from the results of the queries he makes. Security against an adaptive-chosen-target adversary, which is captured below, is the stronger notion of security than the non-adaptive one. It is also stronger than the selective-node security defined in the fs-PKE scheme by Canetti *et al.* [10].
Chosen-ciphertext Security (CCA2): We say an fs-HIBE scheme is semantically secure against adaptive chosen ciphertext, time period, and identity attack, if no polynomial time bounded adversary $\mathcal{A}$ has a non-negligible advantage against the challenger in the following game.

Setup: The challenger takes a security parameter k, and runs the Root Setup algorithm. It gives the adversary the resulting system parameters $params$. It keeps the root secrets to itself.
Phase 1: The adversary issues queries $q_1, \ldots, q_m$, where q_i is one of the followings:

1. Lower-level setup query $(t_i, \text{ID-tuple}_i)$: the challenger runs the LOWER-LEVEL SETUP algorithm to generate the private key $SK_{(t_i,\ \text{ID-tuple}_i)}$ corresponding to $(t_i, \text{ID-tuple}_i)$, and sends $SK_{(t_i,\text{ID-tuple}_i)}$ to the adversary.

2. Decryption query $(t_i, \text{ID-tuple}_i, C_i)$: the challenger runs the LOWER-LEVEL SETUP algorithm to generate the private key $SK_{(t_i,\text{ID-tuple}_i)}$ corresponding to the pair $(t_i, \text{ID-tuple}_i)$, runs the Decryption algorithm to decrypt C_i using $SK_{(t_i,\text{ID-tuple}_i)}$, and sends the resulting plaintext to the adversary.

These queries may be asked adaptively. Also, the queried ID-tuple$_i$ may correspond to a position at any level in the ID hierarchy, and the adversary is allowed to query for a future time and then for a past time.

Challenge: Once the adversary decides that **Phase 1** is over, it outputs two equal length plaintexts $M_0, M_1 \in \mathcal{M}$, a time period t^* and an ID-tuple* on which it wishes to be challenged. The constraint is that no lower-level setup query has been issued for ID-tuple* or any of its ancestors for any time $t \leq t^*$.

The challenger picks a random bit $b \in \{0, 1\}$, and sets $C^* = \text{ENCRYPT}(params, t^*, \text{ID-tuple}^*, M_b)$. It sends C^* as a challenge to the adversary.

Phase 2: The adversary issues more queries $q_{m+1}, \ldots, q_n$, where q_i is one of:

1. Lower-level setup query $(t_i, \text{ID-tuple}_i)$, where the time period t_i and ID-tuple$_i$ are under the same restriction as in **Challenge**: the challenger responds as in **Phase 1**.

2. Decryption query $(t_i, \text{ID-tuple}_i, C_i)$ being not the same as $(t^*, \text{ID-tuple}^*, C^*)$: the challenger responds as in **Phase 1**.

Guess: The adversary outputs a guess $b' \in \{0, 1\}$. The adversary wins the game if $b = b'$. We define its advantage in attacking the scheme to be $|\Pr[b = b'] - \frac{1}{2}|$.

3. A FORWARD-SECURE HIBE SCHEME

Here, we present a forward-secure hierarchical identity-based encryption scheme. Following the presentation standard in the IBE literature [8, 18], we first present an fs-HIBE with *one-way security*. One-way security is the weakest notion of security. It means that it is hard to recover a plaintext with a passive attack. A standard technique, due to Fujisaki and Okamoto [15], converts one-way security to CCA2 security in the random oracle model. We give our definition of one-way security and the Fujisaki-Okamoto conversion of the one-way secure fs-HIBE in the full paper [32]. Next, we first give the number theoretic assumptions needed in our scheme, and then describe the algorithms in our construction. Due to the page limit, proofs of security of our fs-HIBE scheme are shown in the full version of the paper [32].

3.1 Assumptions

The security of our fs-HIBE scheme is based on the difficulty of the Bilinear Diffie-Hellman (BDH) problem [8]. Let $\mathbb{G}_1$ and $\mathbb{G}_2$ be two cyclic groups of some large prime order q. We write $\mathbb{G}_1$ additively and $\mathbb{G}_2$ multiplicatively. Our schemes make use of a bilinear pairing.

Admissible pairings: Following Boneh and Franklin [8], we call $\hat{e}$ an admissible pairing if $\hat{e} : \mathbb{G}_1 \times \mathbb{G}_1 \to \mathbb{G}_2$ is a map with the following properties:

1. Bilinear: $\hat{e}(aP, bQ) = \hat{e}(P, Q)^{ab}$ for all $P, Q \in \mathbb{G}_1$ and all $a, b \in \mathbb{Z}$.

2. Non-degenerate: The map does not send all pairs in $\mathbb{G}_1 \times \mathbb{G}_1$ to the identity in $\mathbb{G}_2$.

3. Computable: There is an efficient algorithm to compute $\hat{e}(P, Q)$ for any $P, Q \in \mathbb{G}_1$.

We refer the readers to papers by Boneh and Franklin [8] and Boneh and Silverberg [9] for examples and discussions of groups that admit such pairings.

Bilinear Diffie-Hellman (BDH) Parameter Generator: As in IBE [8] scheme, a randomized algorithm $\mathcal{IG}$ is a BDH parameter generator if $\mathcal{IG}$ takes a security parameter $k > 0$, runs in time polynomial in k, and outputs the description of two groups $\mathbb{G}_1$ and $\mathbb{G}_2$ of the same prime order q and the description of an admissible paring $\hat{e} : \mathbb{G}_1 \times \mathbb{G}_1 \to \mathbb{G}_2$.

BDH Problem: As in IBE [8] scheme, given a randomly chosen $P \in \mathbb{G}_1$, as well as aP, bP, and cP (for unknown randomly chosen $a, b, c \in \mathbb{Z}_q$), compute $\hat{e}(P, P)^{abc}$.

For the BDH problem to be hard, $\mathbb{G}_1$ and $\mathbb{G}_2$ must be chosen so that there is no known algorithm for efficiently solving the Diffie-Hellman problem in either $\mathbb{G}_1$ or $\mathbb{G}_2$. Note that if the BDH problem is hard for a paring $\hat{e}$, then it follows that $\hat{e}$ is non-degenerate.

BDH Assumption: As in IBE [8] scheme, we say a BDH parameter generator $\mathcal{IG}$ satisfies the BDH assumption if the following is negligible in k for all PPT algorithm $\mathcal{A}$:
$\Pr[(\mathbb{G}_1, \mathbb{G}_2, \hat{e}) \leftarrow \mathcal{IG}(1^k); P \leftarrow \mathbb{G}_1; a, b, c \leftarrow \mathbb{Z}_q :$
$\mathcal{A}(\mathbb{G}_1, \mathbb{G}_2, \hat{e}, P, aP, bP, cP) = \hat{e}(P, P)^{abc}]$.

3.2 fs-HIBE: Implementation

For simplicity of description, our fs-HIBE construction makes use of a version of fs-PKE scheme due to Katz [23]. In Katz's scheme, time periods are associated with the *leaf* nodes of a binary tree (Rather than with all tree nodes as in the scheme by Canetti *et al.* [10]. Our fs-HIBE scheme can also be realized based on the fs-PKE scheme by Canetti *et al.*, which will give faster key update time. The complexity discussion of our scheme is in Section 6).

We construct a full binary tree of height l, as in Katz's scheme [23]. The root of this tree is labeled ϵ; all other nodes are recursively labeled as follows: if the label of a node is w, then its left child is labeled $w0$, and its right child is labeled $w1$. This way, for each time period $i \in \{0, N-1\}$, there is a leaf labeled with the binary representation of i. Following the fs-PKE scheme [10], we denote the k-bit prefix of a word $w = w_1 w_2 \ldots w_d$ by $w|_k$, that is, $w|_k = w_1 \ldots w_k$ for $k \leq d$. The word w is of length d, i.e. $|w| = d$. Let $w|_0 = \epsilon$ and $w = w|_d$.

At the beginning of time, public parameters are generated. At time t, the entity at level h with ID-tuple $(\text{ID}_1, \ldots, \text{ID}_h)$ holds a secret key $SK_{t,(\text{ID}_1,\ldots,\text{ID}_h)}$. Recall that we denote key $SK_{t,(\text{ID}_1,\ldots,\text{ID}_h)}$ as $SK_{t,h}$ when this causes no confusion. $SK_{t,h}$ consists of $(sk_{t,h}, \{sk_{w,h}\})$, where w are all the labels of the right sibling, if one exists, of each ancestor of the node labeled t in the complete binary tree. $sk_{w,h}$ are also called node keys. At the beginning of time, as public parameters are generated, the values $sk_{0,1}$ and $sk_{1,1}$ are created by the root PKG, whose ID is ID_1. Each $sk_{w,h}$, where w is any

string, can be used to compute an $sk_{w,u}$, where $u > h$, for tuple $(\mathrm{ID}_1, \ldots, \mathrm{ID}_u)$ who is a descendant of $(\mathrm{ID}_1, \ldots, \mathrm{ID}_h)$. The algorithm to use is LOWER-LEVEL SETUP.

Each $sk_{w,h}$, where w is any string, is also used to compute the value $sk_{(w \circ b),h}$, where $b = 0$ or 1. They are the child nodes of w on the binary tree. The algorithm for doing so is COMPUTE NEXT, which is defined as follows. In COMPUTE NEXT, an entity (a PKG or a user) with ID-tuple $(\mathrm{ID}_1, \ldots, \mathrm{ID}_h)$ uses the value $sk_{w,h}$ associated with a node w to compute values $sk_{(w0),h}$ and $sk_{(w1),h}$ for the two child nodes of w. COMPUTE NEXT is a helper function and is called by the algorithm ROOT SETUP and UPDATE.

We must delete all information from which $sk_{t',h}$ for $t' < t$ can be inferred. The algorithm for computing the keys for the next time period and erasing old keys is UPDATE.

The public parameters, time t, and ID-tuple $(\mathrm{ID}_1, \ldots, \mathrm{ID}_h)$ are all that a sender needs in order to send an encrypted message to ID-tuple $(\mathrm{ID}_1, \ldots, \mathrm{ID}_h)$ at time t using algorithm ENCRYPT.

The value $sk_{t,h}$ is all that the user with tuple $(\mathrm{ID}_1, \ldots, \mathrm{ID}_h)$ needs in order to decrypt at time t. The algorithm for doing so is DECRYPT.

Let us look at the contents of each $sk_{w,h}$ more closely. It has two components $S_{w,h}$ and $\mathcal{Q}_{w,h}$. $S_{w,h}$ is a point in $\mathbb{G}_1$, and $\mathcal{Q}_{w,h}$ contains a set of Q-values, which will be explained later. If w represents a leaf on the binary tree, $S_{w,h}$ and the Q-values in $\mathcal{Q}_{w,h}$ together are used for decryption for ID-tuple $(\mathrm{ID}_1, \ldots, \mathrm{ID}_h)$ at time w. If w is an internal node on the binary tree, these values are used for generating future decryption keys.

Computing $S_{w,h}$ makes use of $|w| \times h$ number of secret values $s_{w|_k,(\mathrm{ID}_1,\ldots,\mathrm{ID}_j)}$, where $1 \le k \le |w|$ and $1 \le j \le h$. The shorthand notation for $s_{w|_k,(\mathrm{ID}_1,\ldots,\mathrm{ID}_j)}$ is $s_{k,j}$, when $w|_k$ and $(\mathrm{ID}_1, \ldots, \mathrm{ID}_j)$ are clear from the context. Given a node w on the binary tree and an ID-tuple $(\mathrm{ID}_1, \ldots, \mathrm{ID}_h)$, there is a secret value $s_{k,j}$ for every (time, ID-tuple) combination, where time corresponds to a node that has some prefix $w|_k$ of node w (including w itself and excluding the root node ϵ) and ID-tuple is some ancestor $(\mathrm{ID}_1, \ldots, \mathrm{ID}_j)$ of ID-tuple $(\mathrm{ID}_1, \ldots, \mathrm{ID}_h)$ (including itself). Each $s_{k,j}$ is chosen randomly from $\mathbb{Z}_q$.

Each $s_{k,j}$ is also used for computing $Q_{k,j}$, which is called a Q-value. For each $s_{k,j}$ there is a Q-value $Q_{k,j}$, which is an element in $\mathbb{G}_1$. Q-values are used in DECRYPT. $\mathcal{Q}_{w,h}$ is a set of Q-values. All $s_{k,j}$ values are erased once $S_{w,h}$ and Q-values are computed. The construction is shown below.

Construction Let $\mathcal{IG}$ be a BDH parameter generator for which the BDH assumption holds.
ROOT SETUP$(1^k, N = 2^l)$: The root PKG with ID_1 does the following:

1. $\mathcal{IG}$ is run to generate groups $\mathbb{G}_1, \mathbb{G}_2$ of order q and bilinear map $\hat{e}$.

2. A random generator $P \leftarrow \mathbb{G}_1$ is selected along with random $s_\epsilon \leftarrow \mathbb{Z}_q$. Set $Q = s_\epsilon P$.

3. Choose a cryptographic hash function $H_1 : \{0,1\}^* \rightarrow \mathbb{G}_1$. Choose a cryptographic hash function $H_2 : \mathbb{G}_2 \rightarrow \{0,1\}^n$ for some n. The security analysis will treat H_1 and H_2 as random oracles [4]. The message space is $\mathcal{M} = \{0,1\}^n$. The ciphertext space is $\mathcal{C} = \mathbb{G}_1^{l \times h} \times \{0,1\}^n$ where h is the level of the recipient. The system parameters are $params = (\mathbb{G}_1, \mathbb{G}_2, \hat{e}, P, Q, H_1, H_2)$. All operations of fs-HIBE are performed under *params*. The master key is $s_\epsilon \in \mathbb{Z}_q$.

   The root PKG needs to generate not only the sk key associated with the current time period 0, but also the sk keys corresponding to the internal nodes on the binary tree whose bit representations are all 0 except the last bit. The sk key for time 0 is denoted as $sk_{0^l,1}$. The rest of sk values are used by the root PKG to generate keys for future time periods, and are represented as $\{sk_{1,0}, sk_{(01),1}, \ldots, sk_{(0^{l-1}1),1}\}$.

   These values are generated recursively as follows.

   (a) Set the secret point $S_{0,1}$ to $s_\epsilon H_1(0 \circ \mathrm{ID}_1)$, and $S_{1,1}$ to $s_\epsilon H_1(1 \circ \mathrm{ID}_1)$.

   (b) Set secret key $sk_{0,1} = (S_{0,1}, \emptyset)$ and $sk_{1,1} = (S_{1,1}, \emptyset)$. Root PKG uses $sk_{0,1}$ to recursively call algorithm COMPUTE NEXT (defined below) to generate its secret keys. Let $(sk_{w00,1}, sk_{w01,1}) =$ COMPUTE NEXT$(sk_{w0,1}, w0, \mathrm{ID}_1)$, for all $1 \le |w0| \le l-1$.

   (c) Set the root PKG's secret key for time period 0 as $SK_{0,1} = (sk_{0^l,1}, \{sk_{1,1}, sk_{(01),1}, \ldots, sk_{(0^{l-1}1),1}\})$, and erase all other information.

COMPUTE NEXT$(sk_{w,h}, w, (\mathrm{ID}_1 \ldots \mathrm{ID}_h))$: This is a helper method and is called by the ROOT SETUP and UPDATE algorithms. It takes a secret key $sk_{w,h}$, a node w, and an ID-tuple, and outputs keys $sk_{(w0),h}$, $sk_{(w1),h}$ for time nodes $w0$ and $w1$ of $(\mathrm{ID}_1 \ldots \mathrm{ID}_h)$. (Note that this is similar to the EXTRACTION algorithms in the Gentry-Silverberg HIBE and the DERIVATION algorithm in the fs-PKE schemes, only here we extract keys corresponding to nodes in the time hierarchy for a given ID-tuple.)

1. Parse w as $w_1 \ldots w_d$, where $|w| = d$. Parse ID-tuple as $\mathrm{ID}_1, \ldots, \mathrm{ID}_h$. Parse $sk_{w,h}$ associated with time node w as $(S_{w,h}, \mathcal{Q}_{w,h})$, where $S_{w,h} \in \mathbb{G}_1$ and $\mathcal{Q}_{w,h} = \{Q_{k,j}\}$ for all $1 \le k \le d$ and $1 \le j \le h$, except for $k = 1$ and $j = 1$.

2. Choose random $s_{(d+1),j} \in \mathbb{Z}_q$ for all $1 \le j \le h$.

3. Set $S_{(w0),h} = S_{w,h} + \sum_{j=1}^{h} s_{(d+1),j} H_1(w0 \circ \mathrm{ID}_1 \ldots \mathrm{ID}_j)$.

4. Set $S_{(w1),h} = S_{w,h} + \sum_{j=1}^{h} s_{(d+1),j} H_1(w1 \circ \mathrm{ID}_1 \ldots \mathrm{ID}_j)$.

5. Set $Q_{(d+1),j} = s_{(d+1),j} P$ for all $j \in [1, h]$.

6. Set $\mathcal{Q}_{(w0),h}$ and $\mathcal{Q}_{(w1),h}$ to be the union of $\mathcal{Q}_{w,h}$ and $Q_{(d+1),j}$ for all $1 \le j \le h$.

7. Output $sk_{(w0),h} = (S_{(w0),h}, \mathcal{Q}_{(w0),h})$ and $sk_{(w1),h} = (S_{(w1),h}, \mathcal{Q}_{(w1),h})$.

8. Erase $s_{(d+1),j}$ for all $1 \le j \le h$.

LOWER-LEVEL SETUP$(SK_{i,(h-1)}, i, (\mathrm{ID}_1 \ldots \mathrm{ID}_h))$: Let E_h be an entity that joins the hierarchy during the time period $i < N - 1$ with ID-tuple $(\mathrm{ID}_1, \ldots, \mathrm{ID}_h)$. E_h's parent generates E_h's key $SK_{i,h}$ using its key $SK_{i,(h-1)}$ as follows:

1. Parse i as $i_1 \ldots i_l$ where $l = \log_2 N$. Parse $SK_{i,(h-1)}$ as $(sk_{i,(h-1)}, \{sk_{(i|_{k-1}1),(h-1)}\}_{i_k=0})$.

2. For each value $sk_{w,(h-1)}$ in $SK_{i,(h-1)}$, E_h's parent does the following to generate E_h's key $sk_{w,h}$:

(a) Parse w as $w_1 \dots w_d$, where $d \leq l$, and parse the secret key $sk_{w,(h-1)}$ as $(S_{w,(h-1)}, \mathcal{Q}_{w,(h-1)})$.

(b) Choose random $s_{k,h} \in \mathbb{Z}_q$ for all $1 \leq k \leq d$. Recall that $s_{k,j}$ is a shorthand for $s_{w|_k,(\mathrm{ID}_1 \dots \mathrm{ID}_j)}$ associated with time node $w|_k$ and tuple $(\mathrm{ID}_1 \dots \mathrm{ID}_j)$.

(c) Set the child entity E_h's secret point $S_{w,h} = S_{w,(h-1)} + \sum_{k=1}^{d} s_{k,h} H_1(w|_k \circ \mathrm{ID}_1 \dots \mathrm{ID}_h)$.

(d) Set $Q_{k,h} = s_{k,h} P$ for all $1 \leq k \leq d$. Let $\mathcal{Q}_{w,h}$ be the union of $\mathcal{Q}_{w,(h-1)}$ and $Q_{k,h}$ for all $1 \leq k \leq d$.

(e) Set $sk_{w,h}$ to be $(S_{w,h}, \mathcal{Q}_{w,h})$.

3. E_h's parent sets E_h's $SK_{i,h} = (sk_{i,h}, \{sk_{(i|_{k-1}1),h}\}_{i_k=0})$, and erases all other information.

UPDATE$(SK_{i,h}, i+1, (\mathrm{ID}_1 \dots \mathrm{ID}_h))$ (where $i < N-1$): At the end of time i, an entity (PKG or individual) with ID-tuple $(\mathrm{ID}_1, \dots, \mathrm{ID}_h)$ does the following to compute its private key for time $i+1$, as in the fs-PKE scheme [10, 23].

1. Parse i as $i_1 \dots i_l$, where $|i| = l$. Parse $SK_{i,h}$ as $(sk_{(i|_l),h}, \{sk_{(i|_{k-1}1),h}\}_{i_k=0})$. Erase $sk_{i|_l,h}$.

2. We distinguish two cases. If $i_l = 0$, simply output the remaining keys as the key $SK_{(i+1),h}$ for the next period for ID-tuple $(\mathrm{ID}_1, \dots, \mathrm{ID}_h)$. Otherwise, let $\tilde{k}$ be the largest value such that $i_{\tilde{k}} = 0$ (such $\tilde{k}$ must exist since $i < N-1$). Let $i' = i|_{\tilde{k}-1}1$. Using $sk_{i',h}$ (which is included as part of $SK_{i,h}$), recursively apply algorithm COMPUTE NEXT to generate keys $sk_{(i'0^d1),h}$ for all $0 \leq d \leq l - \tilde{k} - 1$, and $sk_{(i'0^{d-\tilde{k}},h)}$. The key $sk_{(i'0^{d-\tilde{k}},h)}$ will be used for decryption in the next time period $i+1$; the rest of sk keys are for computing future keys. Erase $sk_{i',h}$ and output the remaining keys as $SK_{(i+1),h}$.

ENCRYPT$(i, (\mathrm{ID}_1, \dots, \mathrm{ID}_h), M)$ (where $M \in \{0,1\}^n$):

1. Parse i as $i_1 \dots i_l$. Select random $r \leftarrow \mathbb{Z}_q$.

2. Denote $P_{k,j} = H_1(i|_k \circ \mathrm{ID}_1 \dots \mathrm{ID}_j)$ for all $1 \leq k \leq l$ and $1 \leq j \leq h$. Output $\langle i, (\mathrm{ID}_1, \dots, \mathrm{ID}_h), C \rangle$, where $C = (rP, rP_{2,1}, \dots, rP_{l,1}, rP_{1,2}, \dots, rP_{l,2}, \dots, rP_{1,h}, \dots, rP_{l,h}, M \oplus H_2(\hat{e}(Q, H_1(i_1 \circ \mathrm{ID}_1))^r))$.

DECRYPT$(i, (\mathrm{ID}_1, \dots, \mathrm{ID}_h), SK_{i,h}, C)$:

1. Parse i as $i_1 \dots i_l$. Parse $SK_{i,h}$ associated with the ID-tuple as $(sk_{i,h}, \{sk_{(i|_{k-1}1),h}\}_{i_k=0})$ and the key $sk_{i,h}$ as $(S_{i,h}, \mathcal{Q}_{i,h})$. Parse $\mathcal{Q}_{i,h}$ as $\{Q_{k,j}\}$ for all $1 \leq k \leq l$ and $1 \leq j \leq h$, except for $k = 1$ and $j = 1$.

2. Parse C as

$$(U_0, U_{2,1}, \dots, U_{l,1}, U_{1,2}, \dots, U_{l,2}, \dots, U_{1,h}, \dots, U_{l,h}, V).$$

3. $M = V \oplus H_2(\frac{\hat{e}(U_0, S_{i,h})}{g})$, where g is:

$$\Pi_{k=1}^{l} \Pi_{j=2}^{h} \hat{e}(Q_{k,j}, U_{k,j}) \Pi_{k=2}^{l} \hat{e}(Q_{k,1}, U_{k,1}).$$

Using Fujisaki-Okamoto padding [15] and the help of random oracles H_3 and H_4, algorithm ENCRYPT and DECRYPT can be converted into ones with chosen ciphertext security, as in BF-IBE [8] and GS-HIBE [18].

Theorem 3.1. *Suppose there is a nonadaptive adversary $\mathcal{A}$ that has advantage ϵ against the one-way secure fs-HIBE scheme for some* fixed *time t and ID-tuple, and that makes $q_{H_2} > 0$ hash queries to the hash function H_2 and a finite number of lower-level setup queries. If the hash functions H_1, H_2 are random oracles, then there is an algorithm $\mathcal{B}$ that solves the BDH in groups generated by $\mathcal{IG}$ with advantage $(\epsilon - \frac{1}{2^n})/q_{H_2}$ and running time $\mathcal{O}(time(\mathcal{A}))$.*

Theorem 3.2. *Suppose there is an adaptive adversary $\mathcal{A}$ that has advantage ϵ against the one-way secure fs-HIBE scheme targeting some time and some ID-tuple at level h, and that makes $q_{H_2} > 0$ hash queries to the hash function H_2 and at most $q_E > 0$ lower-level setup queries. Let $l = \log_2 N$, where N is the total number of time periods. If the hash functions H_1, H_2 are random oracles, then there is an algorithm $\mathcal{B}$ that solves the BDH in groups generated by $\mathcal{IG}$ with advantage $(\epsilon(\frac{h+l}{e(2lq_E+h+l)})^{(h+l)/2} - \frac{1}{2^n})/q_{H_2}$ and running time $\mathcal{O}(time(\mathcal{A}))$.*

In our proofs, we use similar arguments to ones in the proofs of IBE [8] and HIBE [18] schemes to show that the adversary who uses an fs-HIBE adversary to break the BasicPub [8] scheme does not abort with non-negligible probability. Boneh and Franklin showed that BasicPub is (semantically) secure in the random oracle model under the BDH assumption [8]. To show that our fs-HIBE scheme is secure, we give a reduction from breaking fs-HIBE scheme to BasicPub. The details of the proofs are in the full paper [32].

4. FORWARD-SECURE BROADCAST ENCRYPTION

In this section, we show how the fs-HIBE scheme can be used to build a scalable forward-secure public-key broadcast encryption (fs-BE) scheme which is joining-time-oblivious. In what follows, N denotes the total number of time periods, $\mathcal{E}$ denotes the universe of users and $E = |\mathcal{E}|$.

4.1 fs-BE: Syntax

Forward-Secure Broadcast Encryption (fs-BE): An fs-BE scheme is specified by five poly-time algorithms KEYGEN, REG, UPD, ENC, DEC:

KeyGen: The *key generation algorithm*, is a probabilistic algorithm run by the center to set up the parameters of the scheme. KEYGEN takes as input a security parameter k and possibly $r_{\max}$ (where $r_{\max}$ is a revocation threshold, i.e. the maximum number of users that can be revoked). The input also includes the total number E of users in the system and the total number of time periods N. KEYGEN generates the public key PK and the initial master secret key MSK_0.

Reg: The *registration algorithm*, is a probabilistic algorithm run by the center to compute the secret initialization data for a new user. REG takes as input the master secret key MSK_t at time t, the identity u of the user and the current time period $t < N-1$ and outputs the new secret key $USK_{t,u}$.

Upd: The *key update algorithm*, is a deterministic algorithm run by an entity (center or user) to update its own secret key of time t into a new secret key valid for the following time period $t+1$. For a user, UPD takes as input the public key PK, the identity u of a user, the current time period $t < N-1$, and the user's secret key $USK_{t,u}$, and outputs the new user's secret key $USK_{t+1,u}$. For the center,

the algorithm takes as input the public key PK, the current time period $t < N$, and the key MSK_t, and outputs the secret key MSK_{t+1}.
Enc: The *encryption algorithm*, is a probabilistic algorithm that each content provider can use to encrypt messages. ENC takes as input the public key PK, a message M, the current time period t and a set $\mathcal{R}$ of revoked users (with $|\mathcal{R}| \leq r_{\max}$, if a threshold has been specified to the KEYGEN algorithm), and returns the ciphertext C to be broadcast.
Dec: The *decryption algorithm*, is a deterministic algorithm run by each user to recover the content from the broadcast. DEC takes as input the public key PK, the identity u of a user, a time period $t < N$, the user's secret key $USK_{t,u}$ and a ciphertext C, and returns a message M.

An fs-BE scheme should satisfy the following correctness constraint: for any pair (PK, MSK_t) output by the algorithm $\text{KEYGEN}(k, r_{\max}, N, E)$, any $t < N$, any $\mathcal{R} \subseteq \mathcal{E}$, $(|\mathcal{R}| \leq r_{\max})$, any user $u \in \mathcal{E} \setminus \mathcal{R}$ with secret key $USK_{t,u}$ (properly generated for time period t) and any message M, it should hold that: $M = \text{DEC}(PK, u, t, USK_{t,u}, \text{ENC}(PK, M, t, \mathcal{R}))$.

4.2 fs-BE: Security

In fs-BE scheme, if a user leaks his or her secret key and is not revoked by a content provider, the security of subsequent communications broadcasted by such provider is compromised. As a matter of fact, the forward security of broadcast encryption schemes guarantees that this is the *only* case where unauthorized access to the broadcast content may occur. The advantage of the adversary is not significantly improved even if she corrupts multiple users at different time periods. We formalize the security definition of fs-BE below.
Chosen-ciphertext Security: An fs-BE scheme is *forward-secure against chosen-ciphertext attack* if no polynomial time bounded adversary $\mathcal{A}$ has a non-negligible advantage against the challenger in the following game:
Setup: The challenger takes security parameters $k, r_{\max}$, and runs the KEYGEN algorithm, for the specified number of users E and time periods N. It gives the adversary the resulting system public key PK and keeps the initial master secret key MSK_0 secret to itself.
Phase 1: The adversary issues, in any adaptively-chosen order, queries $q_1, \ldots, q_m$, where q_i is one of the followings:

1. Corrupt query (u, t): the challenger runs algorithm $\text{REG}(MSK_t, u, t)$ to generate the private key $USK_{t,u}$ corresponding to user u at time t, and sends $USK_{t,u}$ to the adversary.

2. Decryption query (u, t, C): the challenger first runs the $\text{REG}(MSK_t, u, t)$ algorithm to recover private key $USK_{t,u}$ corresponding to user u at time t, and then runs decryption algorithm $\text{DEC}(PK, u, t, USK_{t,u}, C)$ to decrypt C, and sends the resulting plaintext to the adversary.

Challenge: Once the adversary decides that **Phase 1** is over, it outputs two equal-length plaintexts $M_0, M_1 \in \mathcal{M}$, and a time period t^* on which it wishes to be challenged. The challenger picks a random bit $b \in \{0, 1\}$, and set $C^* = \text{ENC}(PK, M_b, t^*, \mathcal{R}_{t^*})$, where $\mathcal{R}_{t^*} = \{u \mid \mathcal{A}$ asked a query $\text{Corrupt}(u, t)$, for some $t \leq t^*\}$. It sends C^* as a challenge to the adversary.
Phase 2: The adversary issues more queries $q_{m+1}, \ldots, q_n$, where q_i is one of:

1. Corrupt query (u, t): the challenger first checks that either $u \in \mathcal{R}_{t^*}$ or $t > t^*$ and if so, it responds as in **Phase 1**. Notice that if a bound r_{max} was specified in KEYGEN, then the adversary is restricted to corrupt at most r_{max} distinct users via Corrupt queries.

2. Decryption query (u, t, C): the challenger first checks that either $C \neq C^*$ or $u \in \mathcal{R}_{t^*}$ or $t \neq t^*$ and if so, it responds as in **Phase 1**.

Guess: The adversary outputs a guess $b' \in \{0, 1\}$ and wins the game if $b = b'$. We define its advantage in attacking the scheme to be $|\Pr[b = b'] - \frac{1}{2}|$.

4.3 fs-BE: A Construction Based on fs-HIBE

Here, we show how our fs-HIBE scheme can be applied to the construction of the public-key broadcast encryption of [12] to obtain a forward-secure public-key BE scheme. Dodis and Fazio [12] provided a construction that extends the symmetric-key broadcast encryption scheme of Naor *et al.* [27] to the public-key setting, based on any secure HIBE scheme. The construction of [12] also applies to the scheme of Halevy and Shamir [20], that improves upon the work of [27]. The symmetric-key BE scheme of Halevy and Shamir is an instance of the *Subset Cover Framework* [27]. The main idea of the framework is to define a family $\mathcal{S}$ of subsets of the universe $\mathcal{E}$ of users in the system, and to associate each subset with a key, which is made available to all the users belonging to the given subset. To broadcast a message to all the subscribers except those in some set $\mathcal{R}$, a content provider first *covers* the set of privileged users using subsets from the family $\mathcal{S}$. This is done by identifying a partition of $\mathcal{E} \setminus \mathcal{R}$, where all the subsets are elements of $\mathcal{S}$. Then, the provider encrypts the message for all the subsets in that partition. To decrypt, a user $u \notin \mathcal{R}$ first identifies the subset in the partition of $\mathcal{E} \setminus \mathcal{R}$ to which he belongs, and then recovers the corresponding secret keys from his secret information.

In the public-key BE scheme [12], the subsets containing a given user are organized into groups, and a special secret key, *protokey*, is associated with each of these groups. A user only needs to store these protokeys, from which he can derive the actual decryption keys corresponding to all the subsets in the group. Such an organization of the subsets of the family $\mathcal{S}$ produces a hierarchy, in which the leaves are elements of $\mathcal{S}$ and each internal node corresponds to a group of subsets. Using HIBE, a secret key can be associated with each internal node in the hierarchy, and constitutes the protokey for the group corresponding to that internal node.

In order to add forward secrecy in the public-key BE scheme, we essentially apply the fs-HIBE scheme to the above hierarchy. In fs-BE scheme, a protokey is associated with not only a node in the hierarchy, but also with a time period t. In the KEYGEN operation, the center runs the ROOT SETUP algorithm of fs-HIBE to compute its master secret $SK_{0,1}$. This key evolves with time, and is used by the center to compute protokeys for users. In the REG operation, a user joins the broadcast at some time t, and the center uses its current master secret key $SK_{t,1}$ to derive protokeys for the user by running the LOWER-LEVEL SETUP of fs-HIBE. The center and users evolve their secret keys with time autonomously by calling the UPDATE algorithm of fs-HIBE. In the ENC algorithm, a content provider uses the ENCRYPT algorithm of fs-HIBE to encrypt the message not only with respect to the nodes in the hierarchy that repre-

Parameters	fs-HIBE	MHIBE	fs-BE
Key generation time	$\mathcal{O}(h \log N)$	$\mathcal{O}(h^m)$	$\mathcal{O}(\log^3 E \log N)$
Encryption time	$\mathcal{O}(h \log N)$	$\mathcal{O}(h^m)$	$\mathcal{O}(r \log E \log N)$
Decryption time	$\mathcal{O}(h \log N)$	$\mathcal{O}(h^m)$	$\mathcal{O}(r + \log E \log N))$
Key update time	$\mathcal{O}(h)$	N/A	$\mathcal{O}(\log^3 E)$
Ciphertext length	$\mathcal{O}(h \log N)$	$\mathcal{O}(h^m)$	$\mathcal{O}(r \log E \log N)$
Public key size	$\mathcal{O}(h + \log N)$	$\mathcal{O}(hm)$	$\mathcal{O}(r \log E + \log N)$
Secret key size	$\mathcal{O}(h \log N)$	$\mathcal{O}(h^m)$	$\mathcal{O}(\log^3 E \log N)$

Table 1: Dependencies of parameters on the total number N of time periods, the length h of an ID-tuple, the number m of ID-tuples in an identity-set in MHIBE, the total number E of fs-BE users and the number r of actual revoked users in fs-BE.

sents the subsets in the partition of $\mathcal{E} \setminus \mathcal{R}$, but also to the current time t. In the DEC operation, the user first runs the LOWER-LEVEL SETUP algorithm of fs-HIBE to derive the current secret keys from his protokey at time t. These secret keys are used for decryption. The construction of our fs-BE scheme is given in the full version of the paper [32]. We analyze the complexity of fs-BE operations in Section 6.

5. MULTIPLE HIERARCHICAL IDENTITY-BASED ENCRYPTION SCHEME

ID-based cryptographic schemes have been used in complex access control scenarios [21, 29]. In this paper, we generalize the fs-HIBE into a collusion resistant multiple hierarchical ID-based encryption (MHIBE) scheme, where a message can be encrypted under multiple ID-tuples. The applications of MHIBE scheme include secure communications with users having multiple identities.

Motivations for MHIBE In role-based access control systems (RBAC) [28], individuals are assigned roles according to their qualifications, and access decisions are based on roles. Communications to a specific role may need to be protected so that messages can be read only by members of that role. What makes the problem interesting is that the *intersection* of identities is different from the *union* of identities, which implies that a proper scheme should be collusion-resistant: secure even if adversaries with partial roles collude. In other words, it requires that compromising the private keys of individual identities does not compromise the messages encrypted with the intersection of identities. This property cannot be achieved by the broken Scheme III described in Section 1.3, where two separate HIBE schemes are used, as it is not collusion-resistant.

Identity-set and Joining-path-obliviousness In MHIBE, we define an *identity-set* as the set of identities that a user has, each represented as an ID-tuple. For example, Bob's identity-set is {(Hospital, ER, Doctor), (Hospital, School, Manager)}. He obtains his private key from either of his two parents, who have the identity-set {(Hospital, ER), (Hospital, School, Manager)} and {(Hospital, ER, Doctor), (Hospital, School)}, respectively. The highest-level PKG in this example has the identity-set {Hospital, Hospital}. An MHIBE scheme needs to be *joining-path-oblivious*. This means that encryption should be oblivious of the path from which the receiver and his ancestors acquire their private keys. Having the receiver's identity-set is sufficient to encrypt a message.

Generalization Our fs-HIBE scheme naturally gives rise to an MHIBE scheme. In fs-HIBE, a message is encrypted under both an ID-tuple and the current time. This can be viewed as the encryption under two tuples, one being the current time. Therefore, the identities in MHIBE scheme capture a broader sense of meaning. The MHIBE scheme generalized from our fs-HIBE scheme supports dynamic joins and joining-path-oblivious encryption. More importantly, it is collusion-resistant, which cannot be achieved by using multiple separate HIBE [18] schemes. In our MHIBE implementation, a message encrypted under {(Hospital, ER, Doctor), (Hospital, School, Manager)} or {(Hospital, School, Manager), (Hospital, ER, Doctor)} requires different decryption keys. We note that in this scheme, the fact that a user holds the private key corresponding to multiple identities does not imply that he or she has the private key to any subset of identities.

We omit the details of MHIBE scheme (definition of security, description of scheme, and proof of security), as this is a direct generalization of fs-HIBE scheme. The complexities of various parameters of our MHIBE scheme are shown in Table 1 in Section 6.

6. DISCUSSIONS

We analyze the complexity of our fs-HIBE scheme, the generalized MHIBE scheme, and the fs-BE scheme in Table 1 showing running time complexities and key sizes. Key generation time of fs-HIBE and MHIBE is the time to generate secret keys for a child node by the parent. Key generation time of fs-BE scheme is the running time of REG algorithm. In our fs-HIBE scheme, the time periods correspond to leaf nodes of a binary tree, and the key update time is $\mathcal{O}(h \log N)$, where N is the total number of time periods and h is the length of an ID-tuple. Because of the node arrangement, the key generation time and key update time of our fs-HIBE scheme grows logarithmically with the total number of time periods N. Faster key update time ($\mathcal{O}(h)$) can be achieved, if the time periods are associated with *all* the nodes of the tree in a pre-order traversal, as in the fs-PKE scheme by Canetti *et al.* [10]. Because the realization of such an fs-HIBE scheme can be easily derived from the construction in Section 3.2, it is omitted in this paper. We show the optimized running time in Table 1.

Even dropping the joining-time-obliviousness requirement (as in the naive Scheme II of Section 1.3), our implementation cannot achieve a ciphertext with linear length $\mathcal{O}(h + \log N)$. It remains an interesting open question whether a general fs-HIBE scheme with linear complexity can be realized.

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